Review of the PQCD approach to exclusive B decays

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I review important aspects of the perturbative QCD approach to exclusive B meson decays, concentrating on factorization theorem, gauge invariance, end-point singularities, k_T and threshold resummations, power counting, penguin enhancement, and CP asymmetries.

1 Introduction

Perturbative QCD (PQCD) seems to be a successful approach to exclusive B meson decays. In this talk I will briefly review important aspects of this approach, concentrating on factorization of infrared divergences, gauge invariance of meson light-cone distribution amplitudes, the smearing of end-point singularities, Sudakov suppression from k_T and threshold resummations, power counting of various topologies of diagrams for hadronic B meson decays, dynamical enhancement of penguin contributions, large strong phases from annihilation diagrams, and CP asymmetries in $B \to K\pi$ and $\pi\pi$ decays.

2 Factorization theorem

We start with the lowest-order diagram for the $B \to \pi$ form factor in the kinematic region with a fast-recoil pion, which contains a hard gluon exchanged between the b quark and the spectator quark. The spectator quark in the B meson, forming a soft cloud around the heavy b quark, carries momentum of order $\bar{\Lambda} = M_B - m_b$, M_B (m_b) being the B meson (b quark) mass. The spectator quark on the pion side carries momentum of $O(M_B)$ in order to form the fast-moving pion with the u quark produced in the b quark decay. These dramatic different orders of magnitude in momenta explain why the hard gluon is necessary. Based on the above argument, the hard gluon is off-shell by order of $\bar{\Lambda} M_B$. As explored later, this scale, characterizing heavy-to-light decays, is important for developing the PQCD formalism of exclusive B meson decays.

At higher orders, infinitely many gluon exchanges appear. If all these gluons are hard, higher-order contributions are calculable in perturbation theory. Unfortunately, these diagrams generate infrared divergences. There are two types of infrared divergences, soft and collinear. Soft divergences come from the region of a loop momentum l, where all its components vanish:

$$l^\mu = (l^+, l^-, l_T) \sim (\bar{\Lambda}, \bar{\Lambda}, \bar{\Lambda}) \ . \tag{1}$$

Collinear divergences are associated with a massless quark of momentum $P \sim (M_B, 0, 0_T)$. In the collienar region with l parallel to P, the components of l behave like

$$l^\mu \sim (M_B, \bar{\Lambda}^2/M_B, \Lambda) \ . \tag{2}$$

In both regions the invariant mass of the radiated gluon diminishes as $\bar{\Lambda}^2$, and the corresponding loop integrand may diverge like $1/\bar{\Lambda}^4$. As the phase space for loop integration vanishes like $d^4 l \sim \bar{\Lambda}^4$, logarithmic divergences are generated.

It has been proved [1] that the soft divergences in the $B \to \pi$ form factor can be factorized into a light-cone B meson distribution amplitude, and that the collinear divergences can be factorized into a pion distribution amplitude. The remaining finite piece is assigned into a hard part. Factorization of infrared divergences is performed in momentum, spin, and color spaces. Factorization in momentum space means that a hard part does not depend on the loop momentum of a soft or collinear gluon, which has been absorbed into a meson distribution amplitude. Factorization in spin and color spaces means that there are separate fermion and color flows between a hard part and a distribution amplitude, respectively. To achieve these, we rely on the eikonal approximation for loop integrals in leading infrared regions, the insertion of the Fierz identity to separate fermion flows, and the Ward identity to sum up diagrams with different color structures. Under the eikonal approximation, a soft or collinear gluon is detached from the lines in a hard part and in other distribution amplitudes. The Fierz identity decomposes a full amplitude into contributions characterized by different twists. The Ward identity is essential for deriving factorization theorem in a nonabelian gauge theory.

I emphasize that the hard scale $\bar{\Lambda} M_B$ is essential for constructing a gauge invariant B meson distribution amplitude,

$$\phi_B(x) = \frac{1}{\sqrt{2N_c} M_B} \int \frac{dy^-}{2\pi} e^{ixP_B^+ y^-} \langle 0 | \bar{q}(y^-) \frac{\gamma_5 \not{b}}{2}$$

$$\times P \exp\left[-ig\int_0^{y^-} dzn\cdot A(zn)\right]$$
$$\times b_v(0)|B(P_B)\rangle\ , \tag{3}$$

with the B meson momentum P_B, the velocity $v = P_B/M_B$, the light-like vector $n = (0, 1, 0_T)$, the number of colors N_c, the momentum fraction x associated with the spectator quark, and the rescaled b quark field

$$b_v(w) = \exp(iM_B v\cdot w)\frac{\not{v}+I}{2}b(w)\ . \tag{4}$$

The above nonlocal matrix element is gauge invariant because of the presence of the path-ordered Wilson line integral. A careful investigation shows that the $O(\alpha_s^2)$ diagram with the second gluon attaching the hard gluon contributes to this line integral. That is, this diagram contains the soft divergence, which is factorized into ϕ_B. This is possible, only when the hard gluon is off-shell by the intermediate scale $\bar{\Lambda}M_B$, rather than by $\bar{\Lambda}^2$ or M_B^2.

A hard part is calculable in perturbation theory. A meson distribution amplitude, though not calculable, is universal, since it absorbs long-distance dynamics, which is insensitive to a specific decay of the b quark into light quarks with large energy release. The universality of nonperturbative distribution amplitudes is the fundamental concept of PQCD factorization theorem. Because of this universality, one can extract distribution amplitudes from experimental data, and employ them to make model-independent predictions for other processes.

3 End-point singularities

After developing factorization theorem, we calculate the $B \to \pi$ form factor $F^{B\pi}(q^2)$ at large recoil, where q denotes the lepton pair momentum. A difficulty immediately occurs. The lowest-order diagram for the hard part, which was mentioned above, is proportional to $1/(x_1x_2^2)$, x_1 (x_2) being the momentum fraction associated with the spectator quark on the B meson (pion) side. If the pion distribution amplitude vanishes like x_2 as $x_2 \to 0$ (in the leading-twist, *i.e.*, twist-2 case), $F^{B\pi}$ is logarithmically divergent. If the pion distribution amplitude is a constant as $x_2 \to 0$ (in the next-to-leading-twist, *i.e.*, twist-3 case), $F^{B\pi}$ even becomes linearly divergent. These end-point singularities have caused critiques on the perturbative evaluation of the $B \to \pi$ form factor.

Several methods have been proposed to regulate the above end-point singularities. An on-shell b quark propagator has been subtracted from the hard part as $x_2 \to 0$ in [2]. However, this subtraction renders the lepton energy spectrum of the semileptonic decay $B \to \pi l\bar{\nu}$ vanishes as the lepton energy is equal to half of its maximal value. Obviously, this vanishing is unphysical. The subtraction also leads to a value of $F^{B\pi}(0)$, which is much smaller than the expected one 0.3. A lower bound of x_2 of $O(\bar{\Lambda}/M_B)$ has been introduced in [3] to make the convolution integral finite. However, the outcomes depend on this cutoff sensitively, and PQCD loses its predictive power.

A QCD-consistent prescription has been proposed in [4], where parton transverse moemta k_T are retained in internal particle propagators involved in the hard part. The inclusion of k_T certainly brings in large double logarithms $\alpha_s \ln^2(k_T/M_B)$ through radiative corrections. These large logarithms should be resummed in order to improve the perturbative calculation. The k_T resummation [5,6,7] then sets a distribution of k_T, such that the average of k_T^2 is numerically around $\langle k_T^2\rangle \sim \bar{\Lambda}M_B$ for $M_B \sim 5$ GeV. The off-shellness of internal particles then remain of $O(\bar{\Lambda}M_B)$ even in the end-point region, and the end-point singularities are smeared out. This is so-called Sudakov suppression.

Recently, another type of resummation has been observed. It is not difficult to verify that the loop correction to the weak decay vertex produces the double logarithms $\alpha_s \ln^2 x_2$ [8]. These double logarithms can be factored out of the hard part systematically, and grouped into an exclusive quark jet function. If the end-point region is important, these large logarithms need to be resummed [9] in order to improve the perturbative calculation. The threshold resummation [10,11] for the jet function results in Sudakov suppression, which decreases faster than any power of x_2 as $x_2 \to 0$, and removes the end-point singularities [8]. In conclusion, if the PQCD analysis of the $B \to \pi$ form factor is performed self-consistently, there exist no end-point singularities.

The mechanism of Sudakov suppression can be easily understood by regarding a meson as a color dipole. In the region with vanishing k_T and x, the meson possesses a huge extent in the transverse and longitudinal directions, respectively. That is, the meson carries a large color dipole. At fast recoil, this large color dipole, strongly scattered, tends to emit real gluons. However, these emissions are forbidden in an exclusive process with final-state particles specified. As a consequence, contributions to the $B \to \pi l\bar{\nu}$ decay from the region with vanishing k_T and x must be highly suppressed.

4 Power counting

We then discuss two-body hadronic B meson decays, such as $B \to PP$. These modes involve three scales [12,13,14]: the W boson mass M_W, at which the matching conditions of the effective weak Hamiltonian to the full Hamiltonian are defined, the typical scale $t \sim \sqrt{\bar{\Lambda}M_B}$ of a hard part, which reflects the specific dynamics of a decay mode, and the factorization scale $\bar{\Lambda}$, which characterizes infrared divergences. Above the factorization scale,

perturbation theory is reliable, and radiative corrections produce two types of large logarithms: $\ln(M_W/t)$ and $\ln(t/\bar{\Lambda})$. The former are summed by renormalization-group (RG) equations to give the evolution from M_W down to t described by the Wilson coefficients $c(t)$, while the latter are summed to give the evolution from t to $\bar{\Lambda}$. When the argument of a Wilson coefficient $c(\mu)$ is set to $\mu = t$, we have included the logarithmic piece of the vertex corrections to the four fermion operators to all orders. These vertex corrections have been considered in a modified factorization approach [15,16], which retains the scale independence of predictions for hadronic B meson decays. The finite piece, being of higher orders in the PQCD approach, is dropped.

The hard part for hadronic B meson decays contains all possible Feynman diagrams, such as factorizable diagrams, where hard gluons attach the valence quarks in the same meson, and nonfactorizable diagrams, where hard gluons attach the valence quarks in different mesons. The annihilation topology is also included, and classified into factorizable or nonfactorizable one according to the above definitions. Below I will discuss the power counting of these different topologies of diagrams and argue that all of them should be included in the leading-power PQCD analysis.

As explained in Sec. 2, factorizable amplitudes scale like $1/\bar{\Lambda}M_B$, since the end-point singularities do not exist. For a similar reason, each nonfactorizable diagram also scales like $1/\bar{\Lambda}M_B$. However, because of the soft cancellation between a pair of nonfactorizable diagrams, their sum turns out to scale like $1/M_B^2$. I emphasize that it is more appropriate to count the power of each individual diagram, instead of the power of sum of diagrams. In some case factorizable contributions are suppressed by a vanishing Wilson coefficient, such that nonfactorizable contributions become dominant. For example, factorizable internal-W emisson contributions are strongly suppressed by the Wilson coefficient a_2 in the $B \to J/\psi K^{(*)}$ decays [13]. In some case, such as the $B \to D\pi$ decays, there is no soft cancellation between a pair of nonfactorizable diagrams, and nonfactorizable contributions also become important [13].

A folklore for annihilation contributions is that they are negligible compared to emission contributions. The annihilation conributions from the operators $O_{1,2,3,4}$ with the structure $(V-A)(V-A)$ are small because of helicity suppression. Those from the operators $O_{5,6}$ with the structure $(S-P)(S+P)$, though surviving under helicity suppression, are of $O(1/M_B^2)$. This assumption is reasonable, because the hard gluon in an annihilaton diagram is off-shell by $x_2 x_3 M_B^2$, where $x_2 \sim O(1)$ and $x_3 \sim O(1)$ are the momentum fractions associated with the two fast outgoing light mesons. However, this argument applies only to the real part of annihilation contributions, but not to the imaginary part.

To obtain the imaginary part, the internal quark is required to be on mass shell. Its propagator, proportional to

$$\frac{1}{x_2 M_B^2 - k_T^2} \;, \tag{5}$$

then gives

$$x_2 M_B^2 = k_T^2 \sim \bar{\Lambda} M_B \;, \quad x_2 \sim \frac{\bar{\Lambda}}{M_B} \;, \tag{6}$$

after considering Sudakov suppression from k_T resummation. This smaller x_2 implies that the hard gluon off-shell only by $\bar{\Lambda}M_B$ contributes to the imaginary part, and that the imaginary annihilation amplitudes possess a power behavior the same as factorizable emission ones. Note that the suppression from threshold resummation decreases the above estimation by a factor 3 or 4. This is the reason that an annihilation amplitude is almost purely imaginary, and its magnitude is usually few times smaller than the factorizable emission ones.

At last, I argue that two-parton twist-3 distribution amplitudes, though proportional to the ratio m_0/M_B with the mass m_0 related to chiral condensate, should be included. As stated before, the corresponding convolution integral for the $B \to \pi$ form factor is linearly divergent. This integral, regulated in some way with an effective cutoff $x_c \sim \bar{\Lambda}/M_B$, is proportional to the ratio $M_B/\bar{\Lambda}$. Combining the two ratios m_0/M_B and $M_B/\bar{\Lambda}$, contributions from two-parton twist-3 distribution amplitudes are in fact not suppressed by a power of $1/M_B$:

$$\frac{m_0}{M_B}\int_{\bar{\Lambda}/M_B}^{1} \frac{dx_2}{x_2^2} \sim \frac{m_0}{\bar{\Lambda}} \;, \tag{7}$$

and should be taken into account. Hence, it is appropriate to claim that the PQCD formalism is complete at leading power.

5 Phenomenology

Another important phenomenological consequence related to the special scale $t \sim \sqrt{\bar{\Lambda}M_B}$ is the dynamical enhancement of penguin contributions [17]. The RG evolution of the Wilson coefficients $C_{4,6}(t)$ dramatically increase as $t < M_B/2$, while that of $C_{1,2}(t)$ almost remain constant [18]. With this penguin enhancement, the observed branching ratios of the $B \to K\pi$ decays, about three times larger than those of the $B \to \pi\pi$ decays, can be explained for a smaller unitarity angle $\phi_3 < 90^o$. Note that the former are dominated by penguin contributions and the latter are dominated by tree contributions. In the factorization approach [19] and in the QCD

factorization approach [20], it is assumed that factorizable contributions are not calculable. The leading contribution to a hadronic decay amplitude is then expressed as a convolution of a hard part with a form factor and a meson distribution amplitude. In both approaches the only hard scale is M_B and the intermediate scale $\bar{\Lambda}M_B$ can not appear. Therefore, the dynamical enhancement of penguin contributions does not exist. It is then difficult to account for the $B \to K\pi$ data.

To accommodate the $B \to K\pi$ data in the factorization and QCD factorization approaches, one relies on the chiral enhancement by increasng the mass m_0 to an unreasonablly large value $m_0 \sim 4$ GeV [21]. Whether dynamical enhancement or chiral enhancement is responsible for the large $B \to K\pi$ branching ratios can be tested by measuring the $B \to \phi K$ modes. In these modes penguin contributions dominate and the mass m_0 is replaced by the ϕ meson mass $M_\phi \sim 1$ GeV. If the branching ratios of the $B \to \phi K$ decays are around 4×10^{-6} [22], the chiral enhancement may be essential for the $B \to K\pi$ decays. Without including annihilation contributions which depend on an end-point cutoff sensitively, similar values of the $B \to \phi K$ branching ratios have been derived in [23] using the QCD factorization approach. If the branching ratios are around 10×10^{-6} as predicted in the PQCD approach [24], the dynamical enhancement may be essential.

Because of the large imaginary annihilation contributions in the PQCD formalism, the predicted CP asymmetry ($\sim 30\%$) in the $B^0 \to \pi^\pm\pi^\mp$ decays [25] dominates over that ($\sim 5\%$) in the QCD factorization approach and over that ($\sim 10\%$) in the factorization approach [26]. The CP asymmetries in the $B \to K\pi$ decays can also reach 15% [17]. Future measurements of CP asymmetries can distinguish these different approaches to hadronic B meson decays. For numerical results of all the hadronic modes that have been studied in PQCD, refer to Dr. Y.Y. Keum's talk in this workshop [24].

6 Conclusion

In this talk I have briefly reviewed important aspects and phenomenological consequences of the PQCD approach to exclusive B meson decays. I have explained that the end-point singularities do not exist in a self-consistent PQCD formalism because of Sudakov suppression from k_T and threshold resummations. I have emphasized the special characteristic scale $\bar{\Lambda}M_B$, under which a gauge-invaraint B meson light-cone distribution amplitude can be constructed, annihilation diagrams should be included in a leading-power analysis, penguin contributions are dynamically enhanced, and large CP asymmetries in the $B \to K\pi$ and $\pi\pi$ decays have been predicted.

More applications of the PQCD approach to other two-body B meson decays have been discussed in other talks and posters of this workshop.

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